Written re-exam
at the Department of Economics winter 2018-19

Economics of the Environment, Natural Resources and Climate Change

Final re-exam

12 February 2019

(3 hour closed book exam)

Answers only in English

This exam question consists of 7 pages in total, including this front page.

NB: If you fall ill during an examination at Peter Bangs Vej, you must contact an invigilator who will show you how to register and submit a blank exam paper. Then you leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams! You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Exercise 1. Bioenergy, fossil energy and the greenhouse effect (indicative weight: 3/4)

Wood-based biomass is still the most important source of renewable energy. In the official accounts for emission of greenhouse gases, biomass is considered to be carbon neutral, since it is assumed that the CO_2 emitted when biomass is burned is offset by the sequestration of carbon via the natural growth of the biomass in existing forests. For this reason the use of biomass is typically exempt from taxes on CO_2. This exam exercise asks you to analyze whether such a policy is optimal, using a simplified model of the macro economy and climate change with the following notation:

Y = gross output of final goods before damage from climate change

F = input of fossil energy

B = input of bioenergy

E = total input of energy

c = total cost of producing fossil energy

b = total cost of producing bioenergy

y = net output of final goods available for consumption and investment

X = stock of biomass

S = stock of CO_2 accumulated in the atmosphere

D = output lost due to damages from climate change

g = natural growth of the stock of biomass

W = social welfare

ρ = rate of time preference (constant)

t = time

Final goods are produced by using inputs of energy and a fixed factor of production which is not stated explicitly since it is in constant supply. Hence the gross output of final goods is given by the following production function displaying a positive but declining marginal productivity of energy input:

$$Y_t = f(E_t), \qquad f' > 0, \quad f'' < 0. \tag{1}$$

Fossil energy and bioenergy are assumed to the perfect substitutes in production, so

$$E_t = F_t + B_t. \tag{2}$$

The total cost of producing fossil energy is measured in units of the final good, and the marginal production cost is positive and increasing:

$$c_t = c(F_t), \qquad c' > 0, \quad c'' > 0. \tag{3}$$

Similarly, we have the following cost function describing the total cost of producing bioenergy (by cutting timber):

$$b_t = b(B_t), \qquad b' > 0, \quad b'' > 0. \tag{4}$$

Accounting for the costs of energy production and the damage costs of climate change, the net output available for consumption and investment is

$$y_t = Y_t - c_t - b_t - D_t. \tag{5}$$

Reflecting the greenhouse effect, the damage costs of climate change are an increasing function of the stock of carbon accumulated in the atmosphere, with a non-declining marginal damage cost:

$$D_t = D(S_t), \qquad D' > 0, \quad D'' \geq 0. \tag{6}$$

We may choose our units of measurement such that the burning of one unit of energy (fossil energy or bioenergy) emits one unit of CO_2. We may also measure the natural growth of the stock of wood-based biomass (g) in units of carbon sequestered as a result of the growth of trees. For simplicity, we will abstract from the fact that some of the carbon accumulated in the atmosphere gradually disappears over time, since this happens very slowly. The stock of carbon in the atmosphere will then evolve in the following way, where a dot above a variable indicates its rate of change over time:

$$\dot{S}_t = F_t + B_t - g(X_t), \qquad g' \geq 0. \tag{7}$$

The specification of the growth function $g(X_t)$ reflects an assumption that, over the relevant range of values of the biomass stock X_t, the natural growth of the stock of biomass (measured in units of carbon sequestered) increases with the stock, as larger forests are able to absorb more carbon from the atmosphere.

The actual growth of the biomass stock equals the natural growth minus the amount of biomass harvested for energy use:

$$\dot{X}_t = g(X_t) - B_t. \tag{8}$$

The most abundant type of fossil fuel is coal. The existing global coal reserves are so large that a serious scarcity of coal is a very remote prospect. We will therefore neglect the fact that, ultimately, coal is an exhaustible resource. Instead we will treat the fossil energy in our model as a reproducible input factor, so we do not need to account for the evolution of the remaining reserves of coal.

Finally, we will measure social welfare by the present value of the current and future volume of final goods available for consumption and investment. Applying a constant discount rate ρ, this present value is

$$W_0 = \int_0^\infty y_t e^{-\rho t} dt, \qquad \rho > 0. \tag{9}$$

This completes the description of technology and preferences in the economy.

Question 1.1. A benevolent social planner will choose the time path of energy use so as to maximize the social welfare function (9), subject to the constraints given by eqs. (1) through (8), taking the initial values of the stocks of carbon and biomass (S_0 and X_0) as given. Show that the current-value Hamiltonian (H) for the solution to this optimal control problem can be written as follows, where λ and η are the shadow values of S and X, respectively, and where we have dropped the time subscripts for convenience:

$$H = f(F + B) - c(F) - b(B) - D(S) + \lambda [F + B - g(X)] + \eta [g(X) - B]. \tag{10}$$

Indicate the control variables and the state variables in this problem. Is λ a positive or a negative number?

Question 1.2. Derive the first-order conditions for the social planner's optimal choice of the variables F and B. Provide an economic interpretation of these conditions.

Question 1.3. Write down the co-state equations describing the socially optimal evolution of the shadow values of the stocks of atmospheric carbon and biomass, respectively (the equations for $\dot{\lambda}$ and $\dot{\eta}$ along the economy's optimal time path). Show that the equation for $\dot{\lambda}$ implies that

$$\lambda_t = -\int_t^{\infty} D'(S_u)\, e^{-\rho(u-t)} du. \tag{11}$$

(Hint: You may use Leibniz' Rule to prove the result in (11)). Explain the economic intuition for this result.

We will now consider a market economy with the technology and preferences described by eqs. (1) through (9). The market price of energy will be denoted by p and will be measured relative to the price of final goods which is our numeraire good with a price of 1. The production side of the market economy consists of three sectors: A final goods sector, a sector producing fossil-based energy, and a sector producing wood-based bioenergy. There are many identical firms in each sector, so perfect competition prevails throughout the economy. All firms take the climate and thereby the damage cost $D(S)$ as given.

The production technology of the representative final goods producer is given by (1) and (2). In period t, the net cash flow generated by the final goods producer is $Y_t - p_t E_t - D(S_t) = f(F_t + B_t) - p_t (F_t + B_t) - D(S_t)$. The final goods firm chooses its energy inputs so as to maximize the present value V_0^Y of the current and future net cash flows to its owners. When maximizing this present value given by

$$V_0^Y = \int_0^{\infty} [f(F_t + B_t) - p_t (F_t + B_t) - D(S_t)]\, e^{-\rho t} dt, \tag{x}$$

the firm takes the energy price p_t as well as $D(S_t)$ as given.

The representative producer of fossil-based energy sells it output F_t to the final goods sector. The government levies a carbon tax at the rate of τ_t^F per unit of fossil fuel sold, so the net cash flow earned by the fossil fuel producer is $\left(p_t - \tau_t^F\right) F_t - c(F_t)$ per period, where $c(F_t)$ is given by (3). The producer of fossil energy chooses its volume of output F_t with the purpose of maximizing the present value V_0^F of the net cash flows paid out to its owners, taking the energy price and the carbon tax as given. This present value is

$$V_0^F = \int_0^{\infty} \left[\left(p_t - \tau_t^F\right) F_t - c(F_t)\right] e^{-\rho t} dt. \tag{xi}$$

The representative producer of bioenergy likewise sells its output to the final goods sector. The bioenergy firm owns the forest from which it harvests the timber for energy use. As part of its climate policy, the government may choose to pay a "conservation subsidy" at the rate s_t^B per unit of carbon stored in the forest biomass. The government may also levy a carbon tax at the rate τ_t^B per unit of biomass (measured in tons of carbon) harvested from the forest for energy use. In period t the bioenergy firm therefore earns a net cash flow to its owners equal to $\left(p_t - \tau_t^B\right) B_t - b\left(B_t\right) + s_t^B X_t$, where the cost function $b\left(B_t\right)$ is given by (4). The present value V_0^B of this cash flow is

$$V_0^B = \int_0^\infty \left[\left(p_t - \tau_t^B\right) B_t - b\left(B_t\right) + s_t^B X_t\right] e^{-\rho t} dt. \quad \text{(xii)}$$

Question 1.4. Derive the first-order condition for the final goods firm's privately optimal input of energy $E_t = F_t + B_t$. Give an economic interpretation of this first-order condition. (Hints: Note that since F_t and B_t are perfect substitutes, the final goods firm is indifferent between using fossil energy and bioenergy, so the first-order condtion for the use of the two forms of energy is the same. Notice also from (x) that the present value of the firm's cash flow will be maximized if it maximizes its net cash flow in each single time period t, so you do not need to use optimal control theory to find the optimum, since the firm takes $D\left(S_t\right)$ as given).

Question 1.5. Derive the first-order condition for the privately optimal volume of F_t supplied by the producer of fossil energy. Give an economic interpretation of this first-order condition. (Hint: Note from (xi) that the present value of the firm's cash flow will be maximized if it maximizes its net cash flow in each single time period t, so you do not need to use optimal control theory to find the optimum).

Question 1.6. Since the bioenergy firm owns the forest, it has an incentive to account for the impact of its current harvest of timber on the evolution of the forest biomass determined by eq. (8). The firm will therefore wish to maximize its present value (xii) subject to the stock-flow constraint (8), given the pre-determined initial biomass stock X_0. Use optimal control theory to derive the first-order condition for the bioenergy firm's privately optimal choice of the harvest rate B_t (the firm's control variable) as well as the condition for the privately optimal rate of change over time of the shadow value of the firm's biomass stock, denoted by η_t^B. Provide an economic interpretation of the condition for the optimal choice of B_t.

Question 1.7. Use your previous results to derive the values of the tax and subsidy rates τ_t^F, τ_t^B and s_t^B which will ensure that the market economy will follow the first-best socially optimal time path. (Hint: Find the values of τ_t^F, τ_t^B and s_t^B that will make the first-order conditions for the market economy coincide with the social planner's first-order conditions). Explain the economic intuition for your results. Discuss briefly whether it is realistic to expect that the government can implement the optimal set of taxes and subsidies in practice.

Exercise 2. Principles of optimal environmental taxation (indicative weight: 1/4).

(Hint: You may provide purely verbal answers to the questions in this exercise, but you are also welcome to include equations if you find it useful).

Question 2.1: Explain what determines the optimal tax on a polluting good in a first-best setting where the government can use lump-sum taxes and subsidies.

Question 2.2: Explain what determines the optimal tax on a polluting good when the government cannot use lump sum taxes and has to raise a given amount of revenue from distorting indirect taxes on consumer goods.